Phil 184/284: Formal and Informal Epistemology  
Section 4 Handout  
John Wilcox  
Department of Philosophy & Department of Psychology  
Stanford University

Definitions:

- $\text{Con}(S) = \{\text{logical consequences of } S\}$
  1. Example:
     - Consider the set $\{p, \ p \rightarrow q\}$
     - The set entails the propositions $q$ and $\neg p \lor p$ (among others)
       - (Informally) A set entails some proposition $\varphi$ just in case $\varphi$ is true every time the propositions in the set are true
     - Then, $q, \neg p \lor p \in \text{Con}(\{p, \ p \rightarrow q\})$

  - Expansion =
    1. This is (generally) going from suspension of judgment about $P$ to believing $P$  
    2. Definition:
       - $(+)(B + P) = \{\varphi: \{B \cup \{P\}\} \Rightarrow \varphi\}$
       - In English:
         - $B$ expanded by $P$—i.e. $(B + P)$—is the set of all sentences $\varphi$ such that $\varphi$ is entailed by the union of $B$ and $P$ (where the union of two sets is a set containing every member that is in either (or both) of the two sets)
    3. Example:
       - Let $B = \text{con}(\{A \rightarrow B\})$
       - Then, $(B + A) = \{\varphi: \{B \cup \{A\}\} \Rightarrow \varphi\} = \{A \rightarrow B, A, B, \neg A \lor A, ...\}$

- Maximal non-$P$ belief subset $B^*$ of set $B =$
  1. Definition:
     i. $B^* \subseteq B$
        1. (every member of $B^*$ is a member of $B$)
     ii. $P \notin B^*$
        1. ($P$ is not in $B^*$)
     iii. $\neg(\exists B^\$)[(B^* \subseteq B^\$) \& (B^\$ \subseteq B) \& (P \notin B^\$)]
        1. (there is no other set $B^\$ where $B^*$ is a smaller subset of $B^\$, where $B^\$ is a subset of $B$ and where $P$ is not in $B^\$)
  2. Example:
     - $B = \text{con}(\{P, Q\})$
     - You then lose your belief in $Q$
     - $B^* = \text{con}(\{P\})$
       - $B^* \subseteq B$: Every member of $B$ is in $B$ (since any consequence of $P$ is a consequence of $P$ and $Q$
       - $B \notin B^*$: $P$ is not in the set $B^*$
       - $\neg(\exists B^\$)[(B^* \subseteq B^\$) \& (B^\$ \subseteq B) \& (P \notin B^\$)]: there is no other set $B^\$ which
         o 1) is a bigger than $B^*$,  
         o 2) while $B^\$ is still a subset of $B$
         o 3) and $P$ is not in $B^\$

- What if there are two or more maximal subsets?
  1. Example:
\[ B = \text{con}(\{P, Q, R\}) \]

You then learn \(^\neg(P \& Q)\), so you have to first suspend judgment about \((P \& Q)\)
- Note: contraction is (generally) suspending judgments about prior beliefs, not disbelieving something you previously believed, but it is a step toward disbelief/revision

Then, two maximal subsets \(B^*\) are
- \(B^{1*} = \text{con}(\{P, R\})\)
- \(B^{2*} = \text{con}(\{Q, R\})\)

What should your belief set be?

- **Entrenchment** = how useful a sentence is in inquiry and deliberation\(^1\)
- **Contraction** = \(\text{contr}(B \setminus P) = \)
  1. This is (generally) going from belief in \(P\) to suspending judgment about \(P\)
  2. Definition:
     - If there are some sets \(B^*\)'s which are maximal subsets of \(B\) and no other maximal subset is more entrenched,
       - then \((B - P) = \bigcap B^*\)'s\) (i.e. the contraction of \(B\) by \(P\) is the set of all elements that are shared by all the most entrenched maximal subsets)
     - If there are no such sets,
       - then \((B - P) = B\)
  3. Examples:
     - **Example 1:**
       - Recall:
         - \(B = \text{con}(\{P, Q, R\})\)
         - \(B^{1*} = \text{con}(\{P, R\})\)
         - \(B^{2*} = \text{con}(\{Q, R\})\)
       - What is \((B - P)\)?
         - If \(B^{1*}\) is more entrenched than \(B^{2*}\), then \((B - (P \& Q)) = B^{1*} = \text{con}(P, R)\)
         - If \(B^{2*}\) is more entrenched than \(B^{1*}\), then \((B - (P \& Q)) = B^{2*} = \text{con}(\{Q, R\})\)
         - If \(B^{1*}\) and \(B^{2*}\) are equally entrenched, \((B - (P \& Q)) = B^{1*} \cap B^{2*} = \text{con}(\{R\})\)
     - **Example 2:**
       - \(B = \text{con}(\{P, Q, R\})\)
       - What is \((B - (P \lor \neg P))\)?
         - There is no maximal subset (qua belief set) which does not contain \((P \lor \neg P)\), so \((B - (P \lor \neg P)) = B\)

- **Revision** =
  1. This is going from belief in \(P\) to believing not \(P\)
  2. \((B \ast P) = ((B - \neg P) + P)\)

\(^1\)“The fundamental criterion for determining the epistemic entrenchment of a sentence is how useful it is in inquiry and deliberation. Certain pieces of information are more important than others when planning future actions, conducting scientific investigations, or reasoning in general . . . The epistemic entrenchment of a sentence is tied to its explanatory power and its overall informational value within the belief set.”
1) Let $B = Con\{(p, p \& r)\}$. Furthermore, let every maximal subset of $B$ containing $q$ be more entrenched than any maximal subset of $B$ not containing $q$, and let every other maximal subset of $B$ be equally entrenched. State whether each of the following claims is true or false, and explain your answer

a) $r \in B + q$

This is true. $B$ expanded by $q$—i.e. $(B + q)$—is the set of all sentences $\varphi$ such that $\varphi$ is entailed by the union of $B$ and $q$. $r$ is entailed by $B$ since $r \in Con\{(p, p \& r)\}$, so it must also be entailed by $B$ in union with any other set. Hence, $q \in B + Y$.

2) Present two of your own counter-examples to AGM theory. For each example, explain which postulate or postulates it threatens.

a. USE YOUR OWN WORDS AND EXAMPLES!!

- Logical omniscience objection:
  1. Expansion postulate:
     - (+1) $(B + P)$ is fully logical (it is closed under implication—i.e. if you believe a set of propositions, you also believe everything entailed by those propositions)
       - Recall $(+)(B + P) = \{\varphi; \{B \cup \{P\}\} \Rightarrow \varphi\}$
  2. Objection:
     - A fully rational agent does not always believe the consequences of their beliefs
     - For example:
       - A mathematician might believe in basic mathematical facts which entail the Riemann hypothesis despite not believing the hypothesis
     - This kind of objection raises questions about what the intended target domain of the model is (e.g. an ideally rational vs. somewhat actual rational agent)

- Monotonicity objection:
  1. Expansion postulate:
     - (+5) If everything in $B$ is also in $B^*$, then everything in $(B + P)$ is also in $(B^* + P)$
  2. Objection:
     - Rational belief is non-monotonic (i.e. learning more information can cause you to retract a belief or inhibit the acquisition of a belief)
     - This paves the way to a counter-example for (+5):
       - Propositions:
         - $A = 99%$ of Canadians speak English and not French
         - $B =$ Montreal is in Canada
         - $C = 50%$ of Montreal Canadians speak French and not English
         - $P =$ Pierre is a Montreal Canadian
         - $E =$ Pierre speaks English and not French
       - Belief sets:
         - $B = con\{(A, B)\}$
         - $B^* = con\{(A, B, C)\}$
       - Then:
         - $E \in B + P$ and $B^* \subseteq B$, but $E \notin B^* + P$
         - In other words, everything in $B$ is also in $B^*$, but everything in $(B + P)$ is not in $(B^* + P)$ because additional information in $B^*$ stops one from inferring $E$